ODE - 1

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Question

The following question is obtained from textbook: *Calculus - Early Transcendentals* 8th ed. by James Stewart.

> A Bernoulli differential equation (named after James Bernoulli) is of the form
>
> $$\frac{dy}{dx} + P(x)y = Q(x)y^n$$
>
> Observe that, if $n = 0$ or 1, the Bernoulli equation is linear. For other values of n, show that the substitution $u = y^{1-n}$ transforms the Bernoulli equation into the linear equation
>
> $$\frac{du}{dx} + (1-n)P(x)u = (1-n)Q(x)$$

Solution

Observe that if $u = y^{1-n}$, then $\frac{du}{dy} = (1-n)y^{-n}$. Also, the equation $\frac{du}{dx} = \frac{du}{dy} \cdot \frac{dy}{dx}$ will be useful since we need to isolate $\frac{du}{dx}$. Now, substitute in the value of $\frac{du}{dy}$ into the equation for $\frac{du}{dx}$. Then, we can solve for $\frac{dy}{dx}$.

$$\frac{du}{dx} = \frac{du}{dy} \cdot \frac{dy}{dx} = (1-n)y^{-n} \cdot \frac{dy}{dx}$$

$$\therefore \frac{dy}{dx} = \frac{1}{(1-n)y^{-n}} \cdot \frac{du}{dx}$$

Now, substitute in this value of $\frac{dy}{dx}$ into Bernoulli differential equation and continue simplifying.

$$\frac{1}{(1-n)y^{-n}} \cdot \frac{du}{dx} + P(x)y = Q(x)y^n$$

$$\frac{du}{dx} + P(x)y \cdot (1-n)y^{-n} = Q(x)y^n \cdot (1-n)y^{-n}$$

$$\frac{du}{dx} + (1-n)P(x)y^{1-n} = (1-n)Q(x)y^n y^{-n}$$

$$\boxed{\frac{du}{dx} + (1-n)P(x)u = (1-n)Q(x)}$$